

# RECENT ENHANCEMENTS OF RUNNING BEHAVIOUR ANALYSIS

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## ABSTRACT

Computational techniques allow a very efficient and inexpensive analysis of the running behaviour of railway vehicles. In this paper enhancements related to multibody simulation for three fundamental topics are briefly presented: A software environment for the bifurcation analysis of complex simulation models of arbitrary railway vehicles; consideration of wheelsets' and rails' structural elasticity within the analysis of the running behaviour; crosswind stability analysis under consideration of parametric variability and uncertainty by reliability analysis.

*Keywords: bifurcation analysis, hunting motion, nonlinear dynamics, vehicle-track interaction, flexible wheelsets, flexible rails, crosswind stability, reliability analysis.*

## 1. INTRODUCTION

The dramatic improvement of railway vehicle performances in the last years has been made possible among others by the availability of very efficient computational methods for the analysis of their running behaviour. Even if a very high standard seems to have been already reached in this field, the continuous improvement and enhancement of existing tools and simulation models together with new developments are necessary to increase computational efficiency and accuracy, to cope with unresolved questions, to manage new safety questions. This paper addresses some current topics of such developments.

## 2. BIFURCATION ANALYSIS

### 2.1 Bifurcation analysis and railway vehicle dynamics

A prominent feature of nonlinear dynamical systems – like e.g. railway vehicles, for which the contact between wheel and rail is an important source of nonlinearity – is the possible dependence of their long-time behaviour on the initial state. This means that one and the same system may show a multiplicity of quantitatively *and* qualitatively totally different behaviour patterns of its steady state, i.e. for system time  $t \rightarrow \infty$ , even though all the system's parameters remain unchanged. If possible transient processes are neglected, stationary, periodic, quasi-periodic and/or chaotic behaviour has to be expected. A computer aided method for the examination of nonlinear dynamical systems with respect to the influence of one or more system parameters on existence and shape of these potential behaviour patterns is *numerical bifurcation analysis*, see e.g. [1] for an extensive description. This kind of analysis can be performed either by means of a simple parameter variation over time integrations or by means of the more sophisticated direct method of *path-following* or *continuation*. The principle of path-following combines algorithms for the direct computation of a

system's steady state (up to now, robust ones exist only for stationary and periodic solutions) with the continuation of one-dimensional curves in higher dimensional state-parameter spaces.

A crucial aspect within the comprehensive computational design of a railway vehicle's running dynamics concerns its long-time behaviour while running on an ideally straight track. Starting from an (initial) disturbance, the vehicle's motion relative to the track is evaluated after all transient activity has died away, i.e. only its steady state is regarded. Of particular interest is the influence of certain system parameters like the vehicle's velocity or the coefficient(s) of a suspension element on this steady state.

Varying, for example, the vehicle's velocity  $v$ , a typical scenario given as bifurcation diagram in Fig. 1 is as follows: As long as the velocities are small or moderate every initial disturbance dies away rapidly until a stationary equilibrium is attained. But, as soon as the "critical velocity" is reached, the vehicle's long-time behaviour changes abruptly and radically and an initial disturbance may result in the so-called hunting or limit cycle motion, a periodically oscillating motion of the complete vehicle relative to the track that has to be avoided in everyday operations. Detailed analyses have shown, see e.g. [2], that there even exists a velocity range  $v_{nlin} \leq v \leq v_{lin}$ , where the final steady state depends qualitatively on the initial disturbance. Thus, for one and the same velocity of this range just as a stationary steady state a periodic steady state (hunting) has to be expected as well.

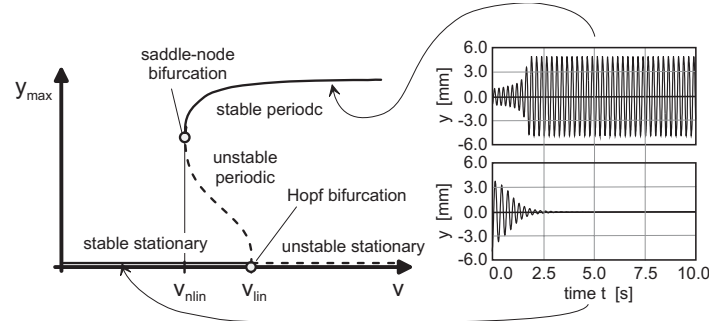


Fig. 1 Typical bifurcation diagram of a railway vehicle. The bifurcation parameter is the velocity  $v$  along the track,  $y_{max}$  is the maximum lateral displacement of a wheelset.

Computational bifurcation analysis is an ideal software tool for examining this kind of dynamic behaviour – with a root locus analysis as a well-known method of linear system analysis only  $v_{lin}$  can be computed as critical velocity! The following section outlines very shortly a software environment for the continuation based bifurcation analysis of arbitrary mechanical systems, a more detailed description can be found in [3, 4]. There, a special emphasis is placed on the application on realistic and therefore necessarily complex simulation models of arbitrary railway vehicles. Algorithms and analyses are restricted to stationary and periodic behaviour.

## 2.2 A software tool for bifurcation analysis of mechanical systems

Regarding a software environment for the robust bifurcation analysis of arbitrary, complex mechanical systems, like e.g. railway vehicles, three major aspects are of

primary interest: Firstly, the *integration* of bifurcation analysis as an additional analysis tool *into* appropriate *simulation software* enables its efficient application on sophisticated simulation models of arbitrary mechanical systems. For this purpose, a reengineered and substantially enhanced version of PATH and the simulation package SIMPACK have been coupled. PATH is a software tool for the continuation based bifurcation analysis of general dynamical systems and has been created originally at DTU, the Technical University of Denmark, see [5] for a description. SIMPACK is a software package for the simulation of general mechanical systems that is commercially distributed, maintained, and designed by Intec GmbH, Wessling, Germany. It is based on a multibody system (MBS) approach and provides a sophisticated and highly specific module for the simulation of arbitrary railway vehicles meeting arbitrary running scenarios.

Secondly, starting from equations of motion given as system of ordinary differential equations (ODEs), the *direct calculation of periodic solutions or limit cycles* is performed by an adapted kind of shooting method, the Poincaré Map Method. Here, one of the main features with respect to an efficient and robust algorithm is the integrated and synchronous evaluation of the variational (differential) equations, i.e. the derivative of the equations of motion with respect to the initial states.

And, finally, the equations of motion of a multibody system are frequently given as a system of *differential algebraic equations (DAE)* of differential index 3. The basic concept to handle this kind of mathematical description is the local parameterisation of the constraint manifold defined by the algebraic constraint equations. This approach yields a local state space form of the equations of motion, i.e. a minimum ODE system being equivalent to the original DAE system.

Of course, the bifurcation analysis tool's potential range of application is not restricted to the specific case of multibody system dynamics. Decoupled from the MBS-code, PATH's enhanced bifurcation algorithms can be applied on general dynamical systems just as well.

## 2.3 Application on a passenger car

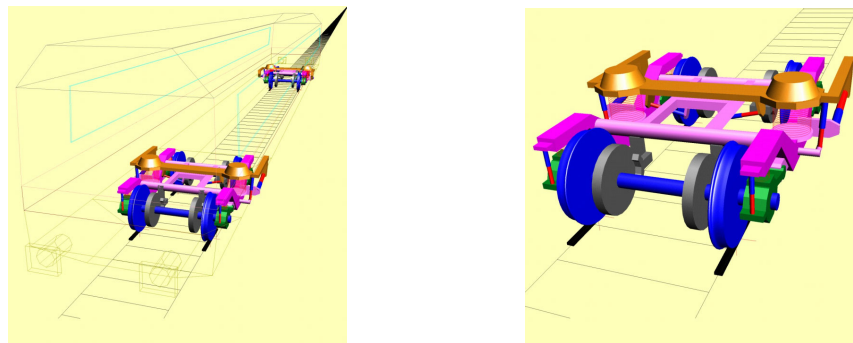


Fig. 2 Simulation model of an *Avmz* passenger car with Fiat 0270 bogies.

The shortly outlined software environment has been applied on a couple of simulation models of railway vehicles showing different degrees of complexity. In what follows, some results of the analyses performed for a 1st class *Avmz* coach's model given in Fig. 2 are presented. The modelling is according to the benchmark

description [6]. The complexity of this model is typical for models used within industry for the extensive computational analysis and design of a railway vehicle's running behaviour.

The mechanical model of the passenger car consists of altogether 15 rigid bodies. The suspension system comprises flexicoil springs with nearly parallel dampers, yaw and lateral dampers, as well as stiff lateral bump stops. All the springs are modelled with constant stiffness while each damper shows an only piecewise linear (thus nonlinear) force-velocity characteristic with a serial stiffness superimposed; hence, the eigendynamics of these damper elements have to be considered, too. The state space of the vehicle model is described by altogether 114 position, velocity, and algebraic coordinates. Hence, a total of 9576 equations has to be integrated for limit cycle calculations (i.e. the nominal system and the variational equations).

Some results of the bifurcation analysis are displayed in Fig. 3. The dynamic long-time behaviour of the vehicle depending on the velocity  $v$  as varied system parameter is represented in these bifurcation diagrams by the maximum lateral deviation of the leading wheelset ( $y_{WS1}$ , left) and of the carbody ( $y_{body}$ , right), each with respect to the track.

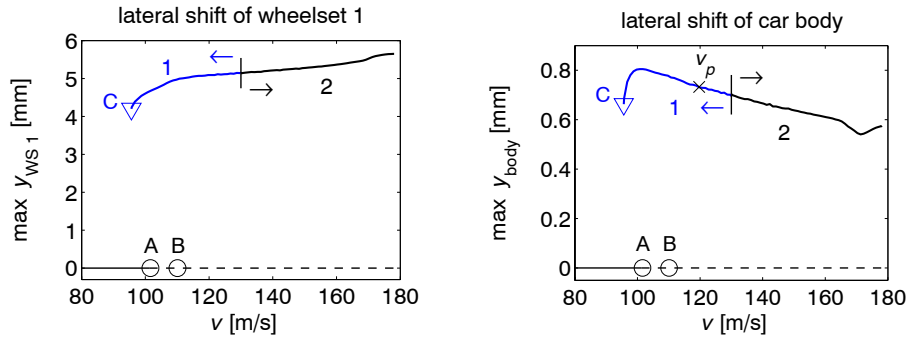


Fig. 3 Numerically computed bifurcation diagram of the *Avmz* passenger car. *Left*: Lateral shift of the leading wheelset. *Right*: Lateral shift of the car body.

The stationary solutions are continued from nearly zero velocity up to the first Hopf bifurcation A, characterised by the velocity  $v_{lin} = v_A = 101.62$  m/s, and beyond, where a second Hopf bifurcation B is detected. Up to now, it is not possible to continue the unstable limit cycle solutions branching off from Hopf bifurcation A. Therefore, the first step to continue limit cycles is to generate an initial estimation of one with the help of a conventional, external time integration. Here, this was done for a velocity  $v = 130.0$  m/s. For decreasing velocities, the path of stable periodic solutions ends in the saddle-node bifurcation C at  $v_{nlin} = v_C = 95.62$  m/s. Since below this limiting velocity no limit cycle oscillations occur, it also represents the critical velocity for this type of vehicle. For increasing velocities, the amplitude of the limit cycle oscillations grows rather slowly due to the nonlinear geometry of wheel and rail profiles and the increasing intensity of the contact between the flanges of the wheels and the rails.

### 3. ELASTIC WHEELSETS AND RAILS

#### 3.1 Motivation

As mentioned before, the wheel-rail contact is a highly nonlinear system which can be very sensitive even to small relative motions between the wheel rim and the rail head. Therefore, the question arises whether small relative motions resulting from elastic deformations of the wheelsets and the rails can have an influence on the wheel-rail contact and, thereby, on the running behaviour of the entire vehicle. In the following, a refined vehicle-track model is presented which takes into account the structural elasticities and dynamics of the wheelsets and the rails, because these are the components next to the wheel-rail contact.

#### 3.2 Modelling

The multibody model of the vehicle represents a passenger coach with two bogies. Here, the car body, the bogie frames and the bolsters are considered as rigid bodies. All bodies are connected by linear spring/damper elements; the only nonlinear force element within the vehicle is the nonlinear yaw damping by dry friction acting between the bolsters and the car body.

The deformations of the wheelsets and the rails are described by a modal synthesis. The required shape functions are obtained by a Finite Element calculation. In both cases, the eigenmodes of the free wheelset and rail, respectively, are applied, because there are no geometrical boundary conditions. For the wheelsets, the eigenmodes of the non-rotating wheelset are used; effects resulting from the rolling motion of the wheelset are taken into account by additional gyroscopic and centrifugal terms, see [7] and [8]. As an example for the deformation of the wheelset, Fig. 4 shows two bending eigenmodes, which correspond to the lowest structural eigenfrequencies. Further informations about the different types of eigenmodes of a wheelset can be found in [9].

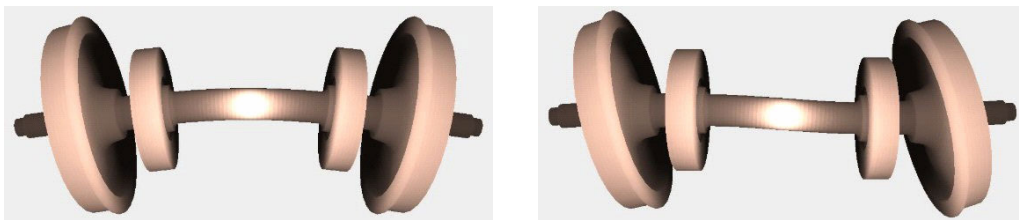


Fig. 4 Bending eigenmodes of the wheelset (*Left*: 84.2 Hz, *Right*: 147.2 Hz)

The rails are connected to the sleepers at the foot, the lateral wheel-rail forces are acting at the rail head. Furthermore, the web of the rail is thin compared to the rail head. Thus, deformations of the rail's cross section can be expected, so that the usual beam theories assuming an invariant shape of the cross section are not sufficient. Therefore, a Finite Element model using prismatic elements for the discretisation is developed. Fig. 5 shows two eigenmodes with distinct deformations of the cross section. The inclination of the rail head can have an impact on the contact geometry.

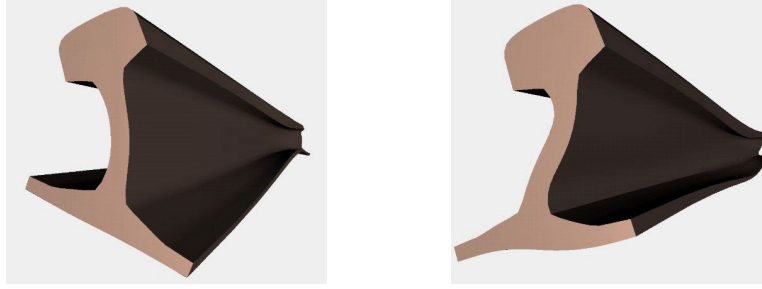


Fig. 5 Eigenmodes of the rail (wavelength: 2.4 m, *left*: 1532.9 Hz, *right*: 4240.9 Hz)

At the ends of the rails, identical boundary conditions are applied, so the track can be considered as a circle, however with neglected curvature. This enables the simulation to be performed without the vehicle reaching an end of the track. This idea of a closed circular structure was already used by Ripke [10].

The wheelsets and the rails are coupled by the nonlinear force element “wheel-rail contact”. The inputs of this force element are the kinematics, i.e. positions and velocities, of the wheel rim and the rail head; the outputs are the forces and torques acting between the wheel and the rail. More details about the coupling between the wheelset and the wheel-rail contact element can be found in [7] and [8].

### 3.3 Calculation Results

To investigate the influence of the structural elasticities, the results obtained with the refined model are compared with those calculated with the “usual” modelling (rigid wheelsets, rigid track elements), see Fig. 6.

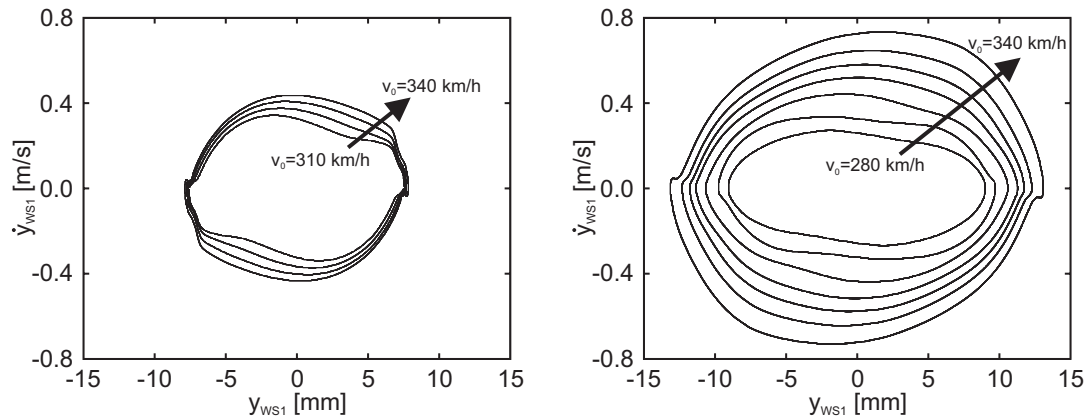


Fig. 6 Phase portraits of the lateral motions of the first wheelset.  
*Left*: Rigid wheelsets, rigid rails; *right*: Elastic wheelsets, elastic rails

The comparison shows three distinct differences:

- ? The amplitudes of the wheelset’s motion are larger for the elastic modelling; this results from deformations of the wheelsets and the rails.
- ? The curves for the elastic wheelset on elastic rails are smoother, while the curves for the rigid wheelset on rigid track elements show distinct kinks at about  $\pm 7$  mm. This kink results from the flange hitting the rail head. Because the wheelsets and the rails themselves are softer than the comparatively stiff wheel-rail contact, the

impact of the flange is cushioned by the structural elasticities, so the kink vanishes. Furthermore, the generalised masses which have to be accelerated by the impact of the flange are reduced.

- ? The curves for  $v_0=280$  km/h,  $v_0=290$  km/h and  $v_0=300$  km/h are missing in the left diagram, because no limit cycle oscillations occur for the rigid wheelsets and rigid track elements at these travelling speeds. This indicates that the structural elasticities cause a drop of the critical speed.

To investigate the influences of the structural elasticities of the wheels on one hand and of the rails on the other hand, the results calculated for the four possible combinations are compared in Fig. 7.

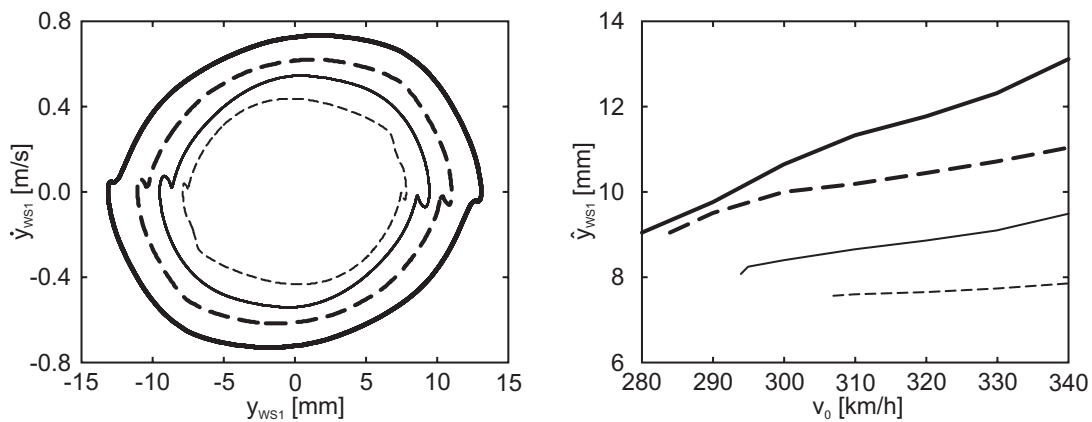


Fig. 7 Phase portraits (*left*) and amplitudes (*right*) of the lateral motion of the first wheelset. Thin/thick line: Rigid/elastic wheelsets, broken/full line: rigid/elastic rails.

The comparison shows that the differences in the running behaviour are mainly caused by the structural elasticities of the wheelset. This elasticity causes the larger drop of the critical speed and the larger increasing of the motion amplitudes. Nevertheless, the influence of the rails' deformations cannot be neglected, even if it is weaker than the impact of the wheelsets' deformations.

## 4. CROSSWIND STABILITY

### 4.1 General remarks

Aerodynamic drag reduction has been a topic since the early years of railway transportation but the analysis of the effects of aerodynamic loads induced by cross winds on the running behaviour of railway vehicles is a relatively new issue. This fact is mainly due to the recent diffusion of lightweight leading cars – as a consequence of EMU solutions – and to the increasing driving velocities. As a matter of fact experience shows that strong cross winds can be a source of critical loads on rolling stock, leading to wheel unloading and finally to vehicle overturn. The originated risk must be accurately evaluated to ensure safety without setting unnecessarily severe design or operational limitations [11].



At present national and international norms for the crosswind stability are being jointly defined or revised in Europe by railway industries, companies and authorities. The safety proof always results from a risk analysis procedure which operates on the outputs of computations describing the driving behaviour of the vehicles: static equilibrium of simplified car models for lower driving velocities, multibody simulation of the transient behaviour of complete car models for higher ones.

In the following sections two important issues of the safety proof are discussed: the role of the wheel-rail contact and the uncertainties in the aerodynamic parameters.

## 4.2 The wheel-rail contact

The most restrictive safety limit for rolling stock in cross winds is due to *overturning*, for which unloading of the wind side wheel of the leading wheelset is taken as an index. The question arises, whether the complete description of the wheel-rail contact is necessary for the safety proof or simplified models are sufficient. The underlying interest for simple models originates from the aim at small computational effort as well as practicability of the norms.

As the most critical scenario for crosswind stability occurs when the wind acts on the vehicle driving on or into a curve, the corresponding non-uniform distribution of lateral and thus vertical forces between the wheelsets of each car can be only reproduced taking into account the radial adjustment of the wheelsets, [12]. Thus, the phenomena at the wheel-rail contact can not be neglected, at least as long as wheel unloading is used to check overturning safety.

For a generic, crosswind stability oriented multibody simulation the wheel-rail contact requires on average about 50% of the CPU-time. Thus, the time saving which can be achieved eliminating the wheel-rail contact is large but not dramatic, because of the very efficient implementation of the wheel-rail module in modern simulation codes like SIMPACK. Furthermore, the mere CPU-time is only a part of the overall effort.

It should be also noted that the use of simplified models for the crosswind stability analysis would require such models to be provided, i.e. derived from the full models. This causes additional effort and increases the complexity of the procedure and of the data management.

## 4.3 Parametric Uncertainty

Each of the six aerodynamic loads  $L$  acting on the vehicle (drag, lift, side forces and roll, yaw, pitch moments) is given by the corresponding static aerodynamic coefficient  $C$ , a characteristic dimension  $D$ , air density  $\rho$ , relative resultant flow velocity  $V_{res}$  and yaw angle  $\varphi$  using the common formula:

$$L = \frac{1}{2} \rho V_{res}^2 C(\varphi) D \quad (1)$$

The aerodynamic coefficients are always handled as fixed deterministic functions of the yaw angle  $\varphi$  but they are actually affected by large uncertainty. In reality they strongly depend on unpredictable environmental conditions (wind turbulence intensity, ground boundary layer, ...) which the vehicle is faced to during real operation. Furthermore they are obtained by static wind tunnel tests on reduced scale models or CFD computations: in both cases systematic errors can not be avoided [13].



Moreover, it is usual to examine the crosswind stability in the very important situation of gusty wind. In this case, on the one hand unsteady aerodynamic phenomena should be taken into account through the aerodynamic admittance function. On the other hand, even if only an ideal gust (e.g. step or ramp) is considered, the gust parameters (amplitude, slope, etc.) can not be reasonably modelled as deterministic, as real wind is a random process.

A way to describe such uncertainties is modelling the affected parameters (aerodynamic coefficients and gust parameters) as stochastic variables. For the resulting problem, namely the risk analysis in presence of uncertainties, many efficient methods can be employed, which are generally denoted as *reliability analysis* methods [14]. Many of these methods were designed already in the second half of the last century for structural reliability analysis; then, in more recent years, new techniques were developed in the fields of robust design. At the moment, the interest on parametric uncertainties is rising again because of the growing use of reliability based optimisation. Such methods enhance not only the accuracy of the risk assessment but also offer a deeper insight in the system behaviour to discover improvement margins in the design [15].

A key point is that the description of the parameters as stochastic variables requires large probabilistic information about them, which is not yet available in the case of crosswind stability. Therefore the reliability analysis must be accompanied by a *probabilistic sensitivity analysis* with respect to the stochastic parameters (distribution type, variance, ...), by which the sensitivity of the system response as well as the sensitivity of the system reliability can be checked [16]. An extreme case, which can also be managed, occurs when only a variability interval is available for a parameter and no assumption about any probabilistic distribution can be met.

## 5. CONCLUSIONS

The computer aided analysis of three fundamental topics related to the running behaviour of railway vehicles – bifurcation analysis for running behaviour, wheelsets' and rails' elasticity, crosswind stability – has been briefly presented. By the use of such computational techniques precious improvements in the efficiency of the design process, in the performances of the final product and in the reliability of operations can be achieved.

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